

Find the slope of the tangent line to the curve  $y = \frac{x^3}{f(x)}$  at the point where  $x = 2$  if  $f(2) = 3$  and  $f'(2) = -5$ . SCORE: \_\_\_\_ / 4 PTS

$$\frac{dy}{dx} = \frac{3x^2 f(x) - x^3 f'(x)}{[f(x)]^2}$$

②

$$\left. \frac{dy}{dx} \right|_{x=2} = \frac{3(2)^2 f(2) - (2)^3 f'(2)}{[f(2)]^2} = \frac{12 \cdot 3 - 8 \cdot (-5)}{3^2} = \frac{76}{9}$$

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Prove the derivative of  $\csc x$  using the definition of the derivative function. Show all steps.

SCORE: \_\_\_\_\_ / 4 PTS

Do NOT use the quotient rule, nor the known derivatives of any other trigonometric functions.

You may use the value of the two limits proved in lecture without proving them again.

$$\frac{d}{dx} \csc x = \lim_{h \rightarrow 0} \frac{\csc(x+h) - \csc x}{h}$$

$$\textcircled{1} = \lim_{h \rightarrow 0} \frac{\frac{1}{\sin(x+h)} - \frac{1}{\sin x}}{h}$$

$$\textcircled{1} = \lim_{h \rightarrow 0} \frac{\sin x - \sin(x+h)}{h \sin(x+h) \sin x}$$

$$\textcircled{1} = \lim_{h \rightarrow 0} \frac{\sin x - \sin x \cosh h - \cos x \sinh h}{h \sin(x+h) \sin x}$$

$$= \lim_{h \rightarrow 0} \left( \sin x \cdot \frac{1 - \cosh h}{h} - \cos x \cdot \frac{\sinh h}{h} \right)$$

$$\lim_{h \rightarrow 0} \sin(x+h) \sin x$$

$$\textcircled{1} = -\sin x \lim_{h \rightarrow 0} \frac{\cosh h - 1}{h} - \cos x \lim_{h \rightarrow 0} \frac{\sinh h}{h}$$

$$\textcircled{1} = -\frac{\cos x}{\sin^2 x} = -\csc x \cot x$$

The position of an object at time  $t$  is given by  $s(t) = e^t \cos t$ .

SCORE: \_\_\_\_\_ / 4 PTS

Find the acceleration of the object at time  $t = \frac{\pi}{3}$ .

$$s'(t) = \boxed{e^t \cos t - e^t \sin t} \quad \textcircled{1}$$

$$s''(t) = \boxed{\cancel{e^t \cos t - e^t \sin t}} - (\cancel{e^t \sin t + e^t \cos t}) \\ = -2e^t \sin t \quad \textcircled{1}$$

$$s''\left(\frac{\pi}{3}\right) = -2e^{\frac{\pi}{3}} \sin \frac{\pi}{3}$$

$$= \boxed{-\sqrt{3}e^{\frac{\pi}{3}}} \quad \textcircled{1}$$

Prove the quotient rule using the definition of the derivative function. Show all steps.

SCORE: \_\_\_\_ / 5 PTS

$$\frac{d}{dx} \left( \frac{f}{g} \right) (x) = \lim_{h \rightarrow 0} \frac{\left( \frac{f}{g} \right)(x+h) - \left( \frac{f}{g} \right)(x)}{h}$$

$$\textcircled{1} = \boxed{\lim_{h \rightarrow 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}}$$

$$\textcircled{1} = \boxed{\lim_{h \rightarrow 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{h g(x+h)g(x)}}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{h g(x+h)g(x)}$$

$$\textcircled{1} = \boxed{\lim_{h \rightarrow 0} \frac{g(x) \cdot \frac{f(x+h) - f(x)}{h} - f(x) \cdot \frac{g(x+h) - g(x)}{h}}{h g(x+h)g(x)}}$$

$$= \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\textcircled{1} = \boxed{\left( g(x) \cdot \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} - f(x) \cdot \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \right) / [g(x)]^2}$$

Find the following derivatives. Simplify all answers appropriately.

SCORE: \_\_\_\_ / 13 PTS

[a]  $\frac{d^3}{dx^3} \frac{4x^3 - 16x}{15\sqrt{x}} = \frac{d^3}{dx^3} \left( \frac{4}{15} x^{\frac{5}{2}} - \frac{16}{15} x^{\frac{1}{2}} \right)$

$$= \frac{d^2}{dx^2} \left( \frac{2}{3} x^{\frac{3}{2}} - \frac{8}{15} x^{-\frac{1}{2}} \right) \textcircled{1}$$

$$= \frac{d}{dx} \left( x^{\frac{1}{2}} + \frac{4}{15} x^{-\frac{3}{2}} \right) \textcircled{12}$$

$$= \left| \frac{1}{2} x^{-\frac{1}{2}} - \frac{2}{5} x^{-\frac{5}{2}} \right| \textcircled{13}$$

[c]  $\frac{d}{dx} (3\pi^x + \frac{1}{e^\pi} - 5x^e)$

$$= \left| 3\pi \times \ln \pi \right| - 5ex^{e-1}$$

\textcircled{2}

\textcircled{2}

\textcircled{1} FOR NOT HAVING  
ANY MORE TERMS  
(I.E.  $\frac{d}{dx} \frac{1}{e^\pi} = 0$ )

[b]  $\frac{d}{dt} \frac{1+3t^2-2t^4}{3-t^4}$  (Your final answer must be one fraction)

$$= \frac{(6t-8t^3)(3-t^4) - (1+3t^2-2t^4)(-4t^3)}{(3-t^4)^2} \textcircled{21}$$

$$= \frac{18t-24t^3-6t^5+8t^7+4t^3+12t^5-8t^7}{(3-t^4)^2}$$

$$= \frac{18t-20t^3+6t^5}{(3-t^4)^2} \textcircled{1}$$

[d]  $\frac{d}{d\theta} (\csc \theta \sec \theta + \cot \theta)$

$$= \left| -\csc \theta \cot \theta \sec \theta + \csc \theta \sec \theta \tan \theta \right|$$

\textcircled{2}

$$= -\frac{1}{\sin \theta} \frac{\cos \theta}{\sin \theta} \frac{1}{\cos \theta} + \frac{1}{\sin \theta} \frac{1}{\cos \theta} \frac{\sin \theta}{\cos \theta} - \csc^2 \theta$$

$$= \left| -2\csc^2 \theta + \sec^2 \theta \right|$$

\textcircled{1}